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A Multi-armed Bandit Approach to Distributed Robust Beamforming in Multicell Networks

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Abstract—This paper addresses the problem of maximizing the weighted signal-to-interference-plus-noise-ratio (SINR) targets at user terminals in a distributed manner in multicell interference networks. The optimization is constrained to strict individual base station (BS) transmit power limitations in the presence of imperfect channel state information (CSI). This problem is numerically intractable due to the coupling effect among a cluster of BSs operating under the same frequency bandwidth and robust constraints that involve the imperfect CSI. We first convert the original problem into a dual total transmit power minimization problem subject to a set of robust SINR constraints in the centralized worst-case scenario. Then the resulting global optimization problem is decomposed into a set of independent subproblems at individual BSs. Finally, a multi-arm bandit based algorithm is proposed to optimally scale the SINR targets in a distributed manner based on individual BS power budgets, and coordinate intercell interference among the BSs with a light inter-BS communication overhead. Simulation results demonstrate the advantage of the proposed scheme in terms of providing larger SINR operation range and robustness to the CSI uncertainties.

I. INTRODUCTION

Intercell interference (ICI) is recognized as a fundamental limiting factor to the system performance for future wireless network that is operated under a shared frequency band. Recently, interference coordination [1], where base stations (BSs) only coordinate at beamforming level for transmission strategies, has shown its advantages in ICI mitigation and inter-BS communications overhead reduction, especially for user terminals (UTs) at cell boundaries [2]. Although the interference coordination significantly relaxes the backhaul link capacity through avoiding UTs' data circulation, it still inflicts a considerable system overhead due to its need to a strict coordinated scheduling to secure the quality of service (QoS) for cell-edge UTs. Therefore, decentralized interference coordination, where only some key intercell coupling parameters are shared among BSs iteratively via inter-BS communications so that the individual BSs can optimize their transmission strategies independently and globally, has attracted the attention of researchers [3], [4]. Assuming perfect knowledge of channel state information (CSI), the authors in [4] propose a distributed iterative subgradient algorithm for QoS and Max-min SINR beamforming design in multicast multicell networks. Also, [5] introduces an outage-constrained distributed beamforming design under individual power constraints with a limited amount of information exchange among BSs. Nevertheless, the authors take no consideration of any channel uncertainties, which is unrealistic and may lead to

unexpected results in practice. On the other hand, acquisition of perfect CSI is essential for BSs to design effective downlink coordinated beamforming. Nevertheless, the QoS control in the multiuser network is limited by the channel uncertainties [6] since it may corrupt the CSI at BSs and may lead to unexpected results in practical system. In general, the CSI perturbations are modeled in two ways: bounded model for quantization errors [7]-[9] and probabilistic model for channel estimation errors [10]-[12]. The authors in [9] investigate the robust worst-case QoS optimization problem in multicell network under the assumption of hyper-sphere bounded CSI errors. They, nevertheless, take no account of individual BS power budgets. Assuming Gaussian distributed CSI error in instantaneous channel, [11] studies an outage constrained robust transmission design via the Bernstein-type inequality method for single cell scenario. Another decentralized approach to a robust power minimization problem is proposed in [12], where a SINR outage threshold is assigned to the QoS constraints. Nevertheless, with a predefined SINR outage probability, the problem may be infeasible for high target SINR in practice.

This paper introduces a robust approach for maximizing the weighted target SINR values at UTs, subject to strict individual BS transmit power limitations in a decentralized manner. This optimization problem is numerically intractable due to the coupling effect among BSs operating under unit frequency bandwidth as well as the fact that the robust SINR constraints that involve the bounded CSI errors have to be satisfied in the intersection of infinite number of convex sets. Hence, we first reformulate the original problem in an equivalent centralized sum-power minimization problem subject to worst-case SINR constraints and transform the intractable problem into a numerically tractable one via the S-Lemma and semidefinite relaxation (SDR) [13]. Then, the multicell-wise general problem is decomposed into a set of equivalent parallel subproblems at individual BSs. Finally, a multi-arm bandit based algorithm is proposed to optimally scale the SINR targets in a distributed manner with a light inter-BS communications overhead based on individual BS power budgets to achieve global optimality across the involved multicells. Simulation results confirm the advantage of the proposed strategy in terms of providing larger SINR operation range and robustness against the CSI uncertainties.

The rest of this paper is organized as follows. Section II introduces the system model and problem formulation. In Section III, the original problem is first reformulated in an

equivalent centralized dual problem. Then, a multi-arm bandit based algorithm is proposed for decoupling the problem into a distributed manner, followed by the signalling overhead load analysis. Simulation results are presented and analyzed in Section IV. Finally, Section V summarizes the paper.

Notations: w , \mathbf{w} , \mathbf{W} , $(\cdot)^H$, $\mathbb{E}(\cdot)$ and $\text{tr}(\cdot)$ present a scalar w , a vector \mathbf{w} , a matrix \mathbf{W} , the complex conjugate transpose operators, the expectation value and trace operators, respectively. $\mathbf{W} \succ 0$ and $\mathbf{W} \succeq 0$ denote that \mathbf{W} is a positive definite and semidefinite matrix, respectively. $[\cdot]_{mn}$ indicates the mn -th element of matrix. The notations $\mathbb{R}^{n \times m}$, $\mathbb{C}^{n \times m}$ and $\mathbb{H}^{n \times m}$ are used for the sets of n -by- m dimensional real matrices, complex matrices and complex Hermitian matrices, respectively. $\text{CN}(\cdot)$ represents complex Gaussian random variables.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a multicell downlink network with a cluster of N_c cells over a shared bandwidth. Each cell consists of one BS equipped with N_t antennas, transmitting to its own K single-antenna UTs. Let BS_i , $i \in \{1, \dots, N_c\}$ and UT_{ik} , $k \in \{1, \dots, K\}$, respectively, represent the i -th BS and the k -th UT in cell i . Also let s_{ik} represent the data symbol for UT_{ik} , $\mathbf{w}_{ik} \in \mathbb{C}^{N_t \times 1}$ denote the associated beamforming vector and $\mathbf{h}_{ijk} \in \mathbb{C}^{N_t \times 1}$ be the channel vector from BS_i to UT_{jk} . Then the signal received by UT_{ik} can be expressed as $z_{ik} = \mathbf{h}_{iik}^H \mathbf{w}_{ik} s_{ik} + \sum_{n \neq k} \mathbf{h}_{iik}^H \mathbf{w}_{in} s_{in} + \sum_{j \neq i} \sum_{m=1}^{N_c} \mathbf{h}_{jik}^H \mathbf{w}_{jm} s_{jm} + n_{ik}$, where the terms in the right hand side represent the desired signal, the total intra-cell interference, the aggregate ICI and the zero-mean circularly symmetric complex Gaussian (ZMCSCG) noise with variance σ_{ik}^2 at UT_{ik} , i.e., $n_{ik} \sim \text{CN}(0, \sigma_{ik}^2)$, respectively. Let $\hat{\mathbf{h}}_{ijk} \in \mathbb{C}^{N_t \times 1}$ and $\mathbf{e}_{ijk} \in \mathbb{C}^{N_t \times 1}$, respectively, denote the estimated channel vector and the corresponding CSI perturbation vector. Then, the true channel vector \mathbf{h}_{ijk} can be modeled as

$$\mathbf{h}_{ijk} = \hat{\mathbf{h}}_{ijk} + \mathbf{e}_{ijk} \quad \forall i, j, k, \quad (1)$$

where CSI errors are assumed to be bounded within an elliptic uncertainty region, i.e., $\mathbf{e}_{ijk}^H \mathbf{C}_{ijk} \mathbf{e}_{ijk} \leq 1$ and $\mathbf{C}_{ijk} \succ 0$ specifies the shape and size of the ellipsoid. Assuming $\mathbb{E}(|s_i|^2) = 1$, the SINR at UT_{ik} can be formulated as

$$\text{SINR}_{ik} = \frac{|\mathbf{h}_{iik}^H \mathbf{w}_{ik}|^2}{\sum_{n \neq k} |\mathbf{h}_{iik}^H \mathbf{w}_{in}|^2 + \sum_{j \neq i} \sum_{m=1}^{N_c} |\mathbf{h}_{jik}^H \mathbf{w}_{jm}|^2 + \sigma_{ik}^2}. \quad (2)$$

Let us consider the robust problem of maximizing the weighted SINR targets at UTs in a multicell network subject to a set of strict upper limits on the transmit power constraints at individual BSs in the presence of CSI errors, as

$$\begin{aligned} \max_{c_i, \mathbf{w}_{ik}, \forall i, k} \quad & c_i \\ \text{s.t.} \quad & \frac{\text{SINR}_{ik}}{\gamma_{ik}} \geq c_i, \quad \forall i, k, \end{aligned} \quad (3a)$$

$$\sum_{k=1}^K \|\mathbf{w}_{ik}\|^2 \leq P_i, \quad \forall i, \quad (3b)$$

$$\mathbf{e}_{ijk}^H \mathbf{C}_{ijk} \mathbf{e}_{ijk} \leq 1, \quad \forall i, j, k, \quad (3c)$$

where γ_{ik} is the requested target SINR by UT_{ik} , P_i represents the transmit power limit at BS i , and c_i indicates the percentage coefficient of the desired SINR targets that can be satisfied at UTs. As the problem in (3) is numerically intractable, we begin by introducing an alternative sum-power minimization problem at individual BSs, as

$$\begin{aligned} \min_{\mathbf{w}_{ik}, \forall k} \quad & f_i(\mathbf{w}_{ik}) \triangleq \sum_{k=1}^K \|\mathbf{w}_{ik}\|^2 \\ \text{s.t.} \quad & \text{SINR}_{ik} \geq c_i \gamma_{ik}, \\ & \mathbf{e}_{ijk}^H \mathbf{C}_{ijk} \mathbf{e}_{ijk} \leq 1, \quad \forall k. \end{aligned} \quad (4)$$

Let $\{\mathbf{w}_{ik}^*\}_k$ be the optimal beamforming vectors for the K UTs in BS_i . Also, let $f_i^* = \sum_{k=1}^K \|\mathbf{w}_{ik}^*\|^2$ represent the optimal objective value for the sum-power at BS_i in problem (4). According to KarushKuhnTucker condition, the optimal beamformers $\{\mathbf{w}_{ik}^*\}_k$ calculated by (3) for any fixed positive real number c_i , satisfy the per BS power constraint in (3b) with equality of $P_i = \sum_{k=1}^K \|\mathbf{w}_{ik}^*\|^2$. In the sequel, we relate the optimal solutions of the problems in (3) and (4) within any cell i through the following Lemma.

Lemma 1. *Solving (3) with (3b) upper bounded by $P_i = f_i^*$ yields an optimal objective value of c_i . Moreover, the optimal objective values of problems (4) and (3) are monotonically non-decreasing and continuous functions of c_i and P_i .*

Proof. See [4]. \square

Therefore, the optimal solution to (3) can be obtained via alternatively solving (4) for a given c_i and searching over different c_i .

III. DISTRIBUTED OPTIMIZATION OF PROBLEM (3)

In this section, we start by introducing a centralized formulation of the sum-power optimization problem in (4) for a given c_i to account for the coupling effects among the multicells. Introducing slack variables $\{p_{ijk}\}_{i,j,k} \in \mathbb{R}$, (4) can be generalized as

$$\begin{aligned} \min_{\mathbf{w}_{ik}, \forall i, k} \quad & \sum_{i=1}^{N_c} \sum_{k=1}^K \|\mathbf{w}_{ik}\|^2 \\ \text{s.t.} \quad & \frac{|\left(\hat{\mathbf{h}}_{iik} + \mathbf{e}_{iik}\right)^H \mathbf{w}_{ik}|^2}{\sum_{n \neq k} |\left(\hat{\mathbf{h}}_{iik} + \mathbf{e}_{iik}\right)^H \mathbf{w}_{in}|^2 + \sum_{l \neq i}^{N_c} p_{lik} + \sigma_{ik}^2} \\ & \geq c_i \gamma_{ik}, \quad \forall i, k, \end{aligned} \quad (5a)$$

$$p_{ijk} \geq \sum_{m=1}^K |\left(\hat{\mathbf{h}}_{ijk} + \mathbf{e}_{ijk}\right)^H \mathbf{w}_{im}|^2, \quad \forall i, j \neq i, k, \quad (5b)$$

$$\mathbf{e}_{ijk}^H \mathbf{C}_{ijk} \mathbf{e}_{ijk} \leq 1, \quad \forall i, j, k, \quad (5c)$$

where p_{ijk} indicates the ICI from BS_{*i*} to UT_{*jk*}. Let the rank-one positive semidefinite matrix be defined as $\mathbf{W}_{ik} = \mathbf{w}_{ik} \mathbf{w}_{ik}^H$, the constraints in (5a) and (5b) can be rewritten as

$$(\hat{\mathbf{h}}_{iik} + \mathbf{e}_{iik})^H \Phi_{ik} (\hat{\mathbf{h}}_{iik} + \mathbf{e}_{iik}) \geq \sum_{l \neq i}^{N_c} p_{lik} + \sigma_{ik}^2, \forall i, k \quad (6)$$

$$p_{ijk} \geq (\hat{\mathbf{h}}_{ijk} + \mathbf{e}_{ijk})^H \Psi_{ijk} (\hat{\mathbf{h}}_{ijk} + \mathbf{e}_{ijk}), \forall i, j \neq i, k, \quad (7)$$

where $\Phi_{ik} = (c_i \gamma_{ik})^{-1} \mathbf{W}_{ik} - \sum_{n \neq k}^K \mathbf{W}_{in}$ and $\Psi_{ijk} = \sum_{m=1}^K \mathbf{W}_{im}$. Hence, the problem (5) can be reformulated as

$$\begin{aligned} \min_{\mathbf{W}_{ik} \succeq 0, \forall i, k} \quad & \sum_{i=1}^{N_c} \sum_{k=1}^K \text{tr}(\mathbf{W}_{ik}) \\ \text{s.t.} \quad & (6) \text{ and } (7), \\ & \mathbf{e}_{ijk}^H \mathbf{C}_{ijk} \mathbf{e}_{ijk} \leq 1, \quad \forall i, j, k \\ & \text{rank}(\mathbf{W}_{ik}) = 1. \end{aligned} \quad (8)$$

The set of non-convex rank-one constraints in problem (8) can be relaxed via SDR approach. However, it is still numerically intractable as the remaining robust constraints that involve CSI errors have to be satisfied in the intersection of infinite number of convex sets. Thus, following the similar principles as in [9], we overcome the intractability via following lemma.

Lemma 2. (*S-Procedure [13]*) *The implication $\mathbf{e}^H \mathbf{A}_1 \mathbf{e} + 2\Re(\mathbf{b}_1^H \mathbf{e}) + d_1 \leq 0 \Rightarrow \mathbf{e}^H \mathbf{A}_2 \mathbf{e} + 2\Re(\mathbf{b}_2^H \mathbf{e}) + d_2 \leq 0$, where $\mathbf{A}_i \in \mathbb{H}^{N_t \times N_t}$, $\mathbf{b}_i \in \mathbb{C}^{N_t}$, $d_i \in \mathbb{R}$ and $\mathbf{e} \in \mathbb{C}^{N_t \times 1}$, holds if and only if there exists a $\mu \geq 0$ such that*

$$\begin{bmatrix} \mathbf{A}_2 & \mathbf{b}_2 \\ \mathbf{b}_2^H & d_2 \end{bmatrix} \preceq \mu \begin{bmatrix} \mathbf{A}_1 & \mathbf{b}_1 \\ \mathbf{b}_1^H & d_1 \end{bmatrix}.$$

We first expand the constraints in (8) in their equivalent quadratic forms of \mathbf{e}_{iik} and \mathbf{e}_{ijk} , respectively, as

$$\mathbf{e}_{iik}^H \mathbf{C}_{iik} \mathbf{e}_{iik} - 1 \leq 0 \Rightarrow \quad (9)$$

$$-\mathbf{e}_{iik}^H \Phi_{ik} \mathbf{e}_{iik} - (\Phi_{ik} \hat{\mathbf{h}}_{iik})^H \mathbf{e}_{iik} - \mathbf{e}_{iik}^H \Phi_{ik} \hat{\mathbf{h}}_{iik} - v_{ik} \leq 0 \quad (10)$$

$$\mathbf{e}_{ijk}^H \mathbf{C}_{ijk} \mathbf{e}_{ijk} - 1 \leq 0 \Rightarrow$$

$$\mathbf{e}_{ijk}^H \Psi_{ijk} \mathbf{e}_{ijk} + (\Psi_{ijk} \hat{\mathbf{h}}_{ijk})^H \mathbf{e}_{ijk} + \mathbf{e}_{ijk}^H \Psi_{ijk} \hat{\mathbf{h}}_{ijk} - v'_{ijk} \leq 0 \quad \forall i, j \neq i, k$$

where $v_{ik} = \hat{\mathbf{h}}_{iik}^H \Phi_{ik} \hat{\mathbf{h}}_{iik} - \sum_{l \neq i}^{N_c} p_{lik} - \sigma_{ik}^2$ and $v'_{ijk} = -\hat{\mathbf{h}}_{ijk}^H \Psi_{ijk} \hat{\mathbf{h}}_{ijk} + p_{ijk}$. Therefore, applying the Lemma 2 to (9) and (10), we can rewrite the optimization problem in (8) into semidefinite programming form with linear matrix inequality

constraints, as

$$\begin{aligned} \min_{\mathbf{W}_{ik} \succeq 0, \forall i, k} \quad & \sum_{i=1}^{N_c} \sum_{k=1}^K \text{tr}(\mathbf{W}_{ik}) \\ \text{s.t.} \quad & \begin{bmatrix} \mu_{ik} \mathbf{C}_{iik} + \Phi_{ik} & \Phi_{ik} \hat{\mathbf{h}}_{iik} \\ (\Phi_{ik} \hat{\mathbf{h}}_{iik})^H & -\mu_{ik} + v_{ik} \end{bmatrix} \succeq 0, \\ & \mu_{ik} \geq 0, \quad \forall i, k, \\ & \begin{bmatrix} \mu_{ijk} \mathbf{C}_{ijk} - \Psi_{ijk} & -\Psi_{ijk} \hat{\mathbf{h}}_{ijk} \\ (-\Psi_{ijk} \hat{\mathbf{h}}_{ijk})^H & -\mu_{ijk} + v'_{ijk} \end{bmatrix} \succeq 0, \\ & \mu_{ijk} \geq 0, \quad \forall i, j \neq i, k, \end{aligned} \quad (11)$$

where the set of auxiliary parameters $\mu_{ik} \geq 0$ and $\mu_{ijk} \geq 0$ appear as a result of the application of Lemma 2. The problem in (11) is convex now and can be optimally solved in a centralized fashion.

In the sequel, the problem in (11) will be decomposed via primal decomposition. Defining $\mathbf{p} \in \mathbb{R}^{(N_c(N_c-1)K) \times 1}$ as a real-valued vector that contains the global intercell coupling variables, i.e., $\mathbf{p} = [p_{121}, p_{122}, \dots, p_{12K}, \dots, p_{N_c 11}, \dots, p_{N_c N_c-1K}]^T$. Then, we use direction vectors \mathbf{d}_{iik} and $\mathbf{d}_{ijk} \in \{0, 1\}^{(N_c(N_c-1)K) \times 1}$ to extract $\sum_{l \neq i}^{N_c} p_{lik}$ and p_{ijk} from global intercell coupling variable \mathbf{p} , respectively, as

$$\begin{cases} \sum_{l \neq i}^{N_c} p_{lik} = \mathbf{d}_{iik}^T \mathbf{p}, & \forall k, \\ p_{ijk} = \mathbf{d}_{ijk}^T \mathbf{p}, & \forall j \neq i, k. \end{cases} \quad (12)$$

Consequently, for any given \mathbf{p} , we can decompose the problem in (11) into N_c sub-problems at any BS i , as

$$\begin{aligned} \min_{\mathbf{W}_{ik} \succeq 0, \forall k} \quad & f_i(\mathbf{W}_{ik}, \mathbf{p}) \triangleq \sum_{k=1}^K \text{tr}(\mathbf{W}_{ik}) \\ \text{s.t.} \quad & \mathbf{E}_{ik} = \mathbf{E}'_{ik} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{d}_{iik}^T \mathbf{p} \end{bmatrix} \succeq 0, \\ & \mathbf{F}_{ijk} = \mathbf{F}'_{ijk} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{d}_{ijk}^T \mathbf{p} \end{bmatrix} \succeq 0, \end{aligned} \quad (13)$$

$$\mu_{ik} \geq 0, \quad \mu_{ijk} \geq 0, \quad \forall k, j \neq i,$$

where

$$\begin{aligned} \mathbf{E}'_{ik} &= \begin{bmatrix} \mu_{ik} \mathbf{C}_{iik} + \Phi_{ik} & \Phi_{ik} \hat{\mathbf{h}}_{iik} \\ (\Phi_{ik} \hat{\mathbf{h}}_{iik})^H & \hat{\mathbf{h}}_{iik}^H \Phi_{ik} \hat{\mathbf{h}}_{iik} - \sigma_{ik}^2 - \mu_{ik} \end{bmatrix}, \\ \mathbf{F}'_{ijk} &= \begin{bmatrix} \mu_{ijk} \mathbf{C}_{ijk} - \Psi_{ijk} & -\Psi_{ijk} \hat{\mathbf{h}}_{ijk} \\ (-\Psi_{ijk} \hat{\mathbf{h}}_{ijk})^H & -\hat{\mathbf{h}}_{ijk}^H \Psi_{ijk} \hat{\mathbf{h}}_{ijk} - \mu_{ijk} \end{bmatrix}, \end{aligned}$$

and the function $f_i(\mathbf{W}_{ik}, \mathbf{p}) \triangleq \sum_{k=1}^K \text{tr}(\mathbf{W}_{ik})$ in (13) explicitly shows the dependence of f_i on \mathbf{p} . Since the optimal solution \mathbf{w}_{ik}^* is obtained as a function of \mathbf{p} , we introduce an algorithm to iteratively coordinates \mathbf{p} and \mathbf{w}_{ik} , $\forall i, k$, at their globally optimal settings of \mathbf{p}^* and \mathbf{w}_{ik}^* , respectively, to minimize the total power consumption in the multicell network. We start by forming the Lagrangian of the primal

sub-problem in (13), as $L_i = \sum_{k=1}^K \text{tr}(\mathbf{W}_{ik}) - \sum_{k=1}^K \text{tr}(\lambda_{ik} \mathbf{E}_{ik}) - \sum_{j \neq i} \sum_{k=1}^K \text{tr}(\lambda_{ijk} \mathbf{F}_{ijk}) - \beta_{ik} \mu_{ik} - \beta_{ijk} \mu_{ijk}$, where $\lambda_{ik}, \lambda_{ijk} \in \mathbb{H}^{(N_t+1) \times (N_t+1)}$, $\beta_{ik}, \beta_{ijk} \geq 0$ are the Lagrange multipliers. Due to the convexity of (13), strong duality holds [13] and the dual function is given by

$$\ell_i = \inf_{\mathbf{W}_{ik} \succeq 0} L_i = \left(\sum_{k=1}^K [\lambda_{ik}]_{(N_t+1)(N_t+1)} \mathbf{d}_{ik}^T - \sum_{j \neq i} \sum_{k=1}^K [\lambda_{ijk}]_{(N_t+1)(N_t+1)} \mathbf{d}_{ijk}^T \right) \mathbf{p} + \Xi_i \left(\{\lambda_{ik}, \beta_{ik}\}_k, \{\lambda_{ijk}, \beta_{ijk}\}_{j \neq i, k} \right), \quad (14)$$

where $\Xi_i = \inf_{\mathbf{W}_{ik} \succeq 0} \sum_{k=1}^K \text{tr}(\mathbf{W}_{ik}) - \sum_{k=1}^K \text{tr}(\lambda_{ik} \mathbf{E}'_{ik}) - \sum_{j \neq i} \sum_{k=1}^K \text{tr}(\lambda_{ijk} \mathbf{F}'_{ijk})$. Defining $\mathbf{g}_i \in \mathbb{R}^{1 \times (N_c(N_c-1)K)}$ as

$$\mathbf{g}_i = \sum_{k=1}^K [\lambda_{ik}^*]_{(N_t+1)(N_t+1)} \mathbf{d}_{ik}^T - \sum_{j \neq i} \sum_{k=1}^K [\lambda_{ijk}^*]_{(N_t+1)(N_t+1)} \mathbf{d}_{ijk}^T, \quad (15)$$

then we can write

$$f_i^*(\mathbf{W}_{ik}^*, \mathbf{p}) = f_i^*(\mathbf{p}) = \ell_i^*(\mathbf{p}) = \mathbf{g}_i \mathbf{p} + \Xi_i \left(\{\lambda_{ik}^*, \beta_{ik}^*\}_k, \{\lambda_{ijk}^*, \beta_{ijk}^*\}_{j \neq i, k} \right). \quad (16)$$

It can be easily concluded from (16) that for any given $\hat{\mathbf{p}}$, the following inequality holds

$$\begin{aligned} \ell_i^*(\mathbf{p}) &= \mathbf{g}_i \mathbf{p} + \Xi_i \left(\{\lambda_{ik}^*, \beta_{ik}^*\}_k, \{\lambda_{ijk}^*, \beta_{ijk}^*\}_{j \neq i, k} \right) \\ &= \mathbf{g}_i (\mathbf{p} - \hat{\mathbf{p}}) + \mathbf{g}_i \hat{\mathbf{p}} + \Xi_i \left(\{\lambda_{ik}^*, \beta_{ik}^*\}_k, \{\lambda_{ijk}^*, \beta_{ijk}^*\}_{j \neq i, k} \right) \\ &\leq \mathbf{g}_i (\mathbf{p} - \hat{\mathbf{p}}) + \ell_i^*(\hat{\mathbf{p}}). \end{aligned}$$

Hence, \mathbf{g}_i is the subgradient vector of $\ell_i^*(\mathbf{p})$ and $f_i^*(\mathbf{p})$. Following a similar sequence of analysis as for the sub-problem in (13), one can easily verify that the subgradient of the general problem in (11), i.e., $\sum_{i=1}^{N_c} f_i^*(\mathbf{p})$, at a given value of \mathbf{p} , denoted by $\mathbf{g} \in \mathbb{R}^{1 \times (N_c(N_c-1)K)}$, is calculated as

$$\begin{aligned} \mathbf{g} &= \sum_{i=1}^{N_c} \left(\sum_{k=1}^K [\lambda_{ik}^*]_{(N_t+1)(N_t+1)} \mathbf{d}_{ik}^T - \sum_{j \neq i} \sum_{k=1}^K [\lambda_{ijk}^*]_{(N_t+1)(N_t+1)} \mathbf{d}_{ijk}^T \right) = \sum_{i=1}^{N_c} \mathbf{g}_i. \quad (17) \end{aligned}$$

To achieve minimization of total transmit power across the multiple cells while optimally account for the coupling inter-cell effects in a distributed manner, we proceed as follows. At a

given value of c_i , each BS i individually solves its subproblem (13), obtains its subgradient vector \mathbf{g}_i and shares it with other BSs via an inter-BS communications phase. Then, each BS i locally calculates the global subgradient \mathbf{g} as per (17) and updates the global coupling vector \mathbf{p} according to the projected subgradient method, as follows,

$$\mathbf{p}^{[t+1]} = \max \left(0, \mathbf{p}^{[t]} - \frac{\alpha \mathbf{g}^{[t]T}}{\sqrt{t} \|\mathbf{g}^{[t]}\|} \right), \quad (18)$$

where the superscript t denotes the iteration index of inner problem (13) and α represents the step size. Then, each BS i adjusts c_i and scales its target SINR according to the calculated solution of transmit power. The steps are summarized in Algorithm 1. The Algorithm 1 is guaranteed to converge to the optimal solution of (13) provided a proper selection of step size α . If rank of the obtained solutions for \mathbf{W}_{ik} are greater than one, the algorithm converges to suboptimal solutions by approximating the feasible beamforming vectors via the standard Gaussian randomization method [4]. However, since the BSs individually searches for their own c_i without considering other BSs, the obtained c_i may not be global optimum. Consequently, this paper considers searching for the global optimal c_i as a multi-armed bandit (MAB) problem and proposes a learning based upper confidence bound (UCB) algorithm in the sequel to search for the optimal c_i across all BSs in a decentralized fashion.

A. UCB Algorithm for finding the globally optimal c_i

The MAB problem is formulated as a system of N_c arms, each being associated with i.i.d. stochastic rewards. The objective is to maximize the accumulated reward by alternatively acquiring new knowledge, known as exploration, while simultaneously optimizing the decisions based on existing partial knowledge, known as exploitation, in multiple rounds [14]. This paper extracts an abstract idea of MAB problem, where playing an arm at each round is equivalent to running Algorithm 1, i.e., *Exploration for finding reward of the i -th BS*, to estimate the reward for a BS at the n -th round. In the sequel, we introduce the UCB Algorithm, i.e., Algorithm 2, to search for the global optimal c_i at the i -th BS. Due to the fact that the coupling effect among all BSs is negligible for low SINR targets, each BS individually searching for their own c_i barely induces interference to other BSs. Thus, the Algorithm 2 firstly executes coarse tuning to adjust c_i rapidly so that the actual transmit power at each BS lies within the range of minus 30 per cent of the per-BS power limitation. Then, by adopting fine tuning, the BSs alternatively adjust their c_i on the basis of their rewards and interactions. Let $\bar{\mathcal{R}}(\text{BS}_i^{[n]})$ and $\hat{\mathcal{R}}(\text{BS}_i^{[n]})$, respectively, be defined as the estimated mean reward and adjusted reward for the i -th BS at the n -th round. In the n -th round of fine tuning, each BS calculates the estimated mean reward as per Algorithm 1 and the adjusted reward as per Algorithm 2. Then, in the $(n+1)$ -th round, only the BSs with the highest adjusted reward will run the Algorithm 1 to search for a new c_i , while other BSs will maintain the same

c_i as in the previous round. Note that $\sqrt{\frac{3\ln n}{2T_i^{[n]}}}$ in Algorithm 2 reflects the fundamental tradeoff between the exploration that examines the unknown rewards and the exploitation that chooses the good rewards so far, where $T_i^{[n]}$ denotes the total number of times the Algorithm 1 has been runned at the i -th BS in the n -th round. By adjusting the value of $c_i^{[\min]}$ and $c_i^{[\max]}$, one can control the overall system performance conveniently.

Algorithm 1 Exploration for finding reward of the i -th BS

```

1: Initialize:  $t = 0$ ,  $\mathbf{p}(0) \in \mathbb{R}^{K(N_c(N_c-1)+1) \times 1}$ ;
2:  $c_i^{[n]} = (c_i^{[\min]} + c_i^{[\max]})/2$ ;
3: while the inner problem in (13) is not converged do
4:   Solve (13);
5:   Calculate the local subgradient  $\mathbf{g}_i$  using (15);
6:   Exchange  $\mathbf{g}_i$  with the other BSs;
7:   Form the global subgradient as  $\mathbf{g} = \sum_{i=1}^{N_c} \mathbf{g}_i$ ;
8:   Update the global variable  $\mathbf{p}$  according to (18);
9:   Increment the iteration number  $t = t + 1$ ;
10: end while
11:  $\hat{P}_i^{[n]} = f_i^* = \sum_{k=1}^K \text{tr}(\mathbf{W}_{ik}^*)$ ;
12: Calculate estimated mean reward  $\bar{\mathcal{R}}(\text{BS}_i^{[n]}) = P_i - \hat{P}_i^{[n]}$ ;
13: if  $\bar{\mathcal{R}}(\text{BS}_i^{[n]}) \geq 0$ ;
14:   then  $c_i^{[\min]} = c_i^{[n]}$ ;
15: else  $c_i^{[\max]} = c_i^{[n]}$ ;
16: end if

```

Algorithm 2 UCB Algorithm for finding global optimal c_i

```

1: Initialize:  $n = 0$ ,  $\bar{\mathcal{R}}(\text{BS}_i^{[n]}) = \hat{\mathcal{R}}(\text{BS}_i^{[n]}) = 0$ ,  $n_{\max}$ ,  $c_i^{[\min]}$ ,  $c_i^{[\max]}$ ;
2: Coarse tuning: Run Algorithm 1 until  $\hat{P}_i^{[n]} \in [0.7*P_i, P_i]$ ;
3: Fine tuning: While  $n \leq n_{\max}$  do
4:    $n = n + 1$ ;
5:   Calculate the adjusted reward  $\hat{\mathcal{R}}(\text{BS}_i^{[n]}) = \bar{\mathcal{R}}(\text{BS}_i^{[n]}) + \sqrt{\frac{3\ln n}{2T_i^{[n]}}}$ ;
6:   BS $_i$  exchanges  $\hat{\mathcal{R}}(\text{BS}_i^{[n]})$  with other BSs;
7:   if  $\hat{\mathcal{R}}(\text{BS}_i^{[n]}) \geq \hat{\mathcal{R}}(\text{BS}_j^{[n]})$ ,  $\forall j \in N_c, j \neq i$ 
8:     then Run Algorithm 1;
9:   else  $c_i^{[n+1]} = c_i^{[n]}$  and run line 3-11 of Algorithm 1;
10: end while
11: return  $\{\mathbf{w}_{ik}\}_{i,k}$  and  $c_i$ 

```

B. Backhaul Signaling Load Analysis

In this section, the backhaul signaling overhead per iteration of our proposed strategy, the coordinated beamforming design in [15] that requires full CSI to be shared among BSs, and the alternating direction method of multipliers (ADMM) approach in [9] will be analyzed. For the i -th BS in our proposed strategy, the major information that need to be exchanged with the other BSs in each iteration for solving inner problem (13) is the subgradient \mathbf{g}_i that contains $N_c K$ non-zero real-valued entries, i.e., $[\lambda_{ik}^*]_{(N_t+1)(N_t+1)}$, $\forall k$ and $[\lambda_{ijk}^*]_{(N_t+1)(N_t+1)}$, $\forall k, j \neq i$.

The resulting inter-BS communication overhead per iteration for all BSs is $O(N_c^2 K(N_c - 1))$. However, for the full CSI design in [15], the information exchange at each BS is $O(N_c K(N_c - 1))$ of $N_t \times 1$ complex-valued CSI vectors. The total signaling overhead is then $O(4N_t N_c^2 K(N_c - 1))$. Interestingly, ADMM approach in [9] requires each BS to inform other BSs with its $N_c K$ real-valued local ICI variables at each iteration, resulting in a same per iteration backhaul signaling load of $O(N_c^2 K(N_c - 1))$. Thus, the proposed strategy that exchanges only key intercell coupling parameters consumes lighter inter-BS communication overhead as compared to [15].

IV. SIMULATION RESULTS

We consider a cluster of 3 neighbouring cells, each cell consists of one BS equipped with 8 antennas. 2 UTs are randomly dropped in the vicinity of the boundaries in each cell to account for the worst coupling effect amongst BSs induced by ICI. Similar to [15], a correlated channel model is adopted as $\hat{\mathbf{h}}_{ijk} = \mathbf{R}_{ijk}^{1/2} \mathbf{h}_w, \forall i, j, k$, where $\mathbf{h}_w \sim \mathcal{CN}(0, 1) \in \mathbb{C}^{N_t \times 1}$ and $\mathbf{R}_{ijk} \in \mathbb{C}^{N_t \times N_t}$ is the channel covariance matrix. The (m, n) -th element of \mathbf{R}_{ijk} is given by $[\mathbf{R}_{ijk}]_{mn} = e^{j\frac{2\pi\delta}{\lambda}[(n-m)\sin\theta_{ijk}]} e^{-2[\frac{\pi\delta\sigma}{\lambda}(n-m)\cos\theta_{ijk}]^2}$, $m, n \in [1, N_t]$, where δ is the antenna spacing, λ denotes the wavelength of the carrier, θ_{ijk} is the estimated angle of departure and $\sigma = 2^\circ$ is the angular offset standard deviation. In order to count for the path loss, fading and shadowing, we also scale the channel vector $\hat{\mathbf{h}}_{ijk}$ and the corresponding error vector \mathbf{e}_{ijk} by $\sqrt{G_a L_{ijk} \sigma_F^2} e^{-0.5(\frac{\sigma_s \ln 10}{100})^2}$, where $G_a = 15$ dBi denotes the antenna gain, $L_{ijk} = 128.1 + 37.6 \log_{10}(\ell)$, ℓ in km is the path loss between BS $_i$ and UT $_{jk}$, σ_F^2 denotes the variance of the complex Gaussian fading coefficient and $\sigma_s = 10$ dB represents the standard deviation of the log-normal shadowing. Equal noise variance $\sigma_{ik}^2 = -127$ dBm and SINR targets γ_{ik} are used for all UTs and same transmit power restriction $P_i = 30$ dBm is applied to all the BSs. We further assume that the CSI errors are spherically bounded, i.e., $\mathbf{C}_{ijk} = 1/r_e^2 \mathbf{I}$, with uncertainty radius of $r_e = \sqrt{0.05}$ for simplicity. We set $c_i^{[\max]} = 1$ and $c_i^{[\min]} = 0$ in Algorithm 2 to optimize the trade-off between power constraints at individual BSs and desired SINR targets at UTs with minimum SINR outage. Simulation results are obtained and averaged via CVX [16].

Fig. 1 presents the performance comparison of total transmit power for the proposed transmission strategy against other schemes, under strict per-BS transmit power limitation of 30 dBm. The comparative schemes are, respectively, the conventional non-coordinated beamforming scheme, the centralized non-robust beamforming scheme, the centralized robust beamforming scheme in [7] and the distributed robust power minimization scheme in [3]. Note that the x axis represents the target SINR γ_{ik} . As can be observed from the figure, the proposed strategy performs overwhelmingly better than the conventional scheme in terms of achieving higher SINR targets and closely follows its distributed robust counterpart in [3] until the per BS power constraint is attained at 16 dB target SINR. Furthermore, nearly all of the comparative

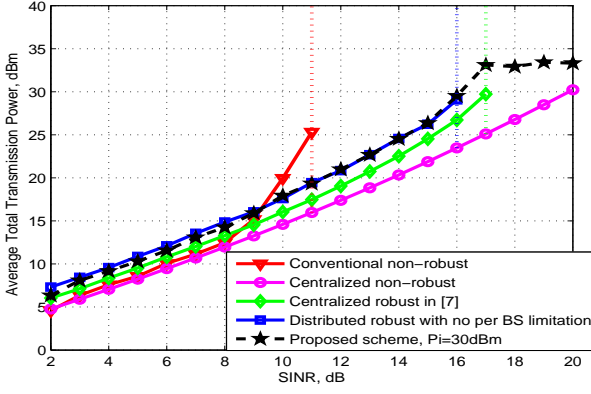


Fig. 1: Comparison of total transmit power for different schemes.

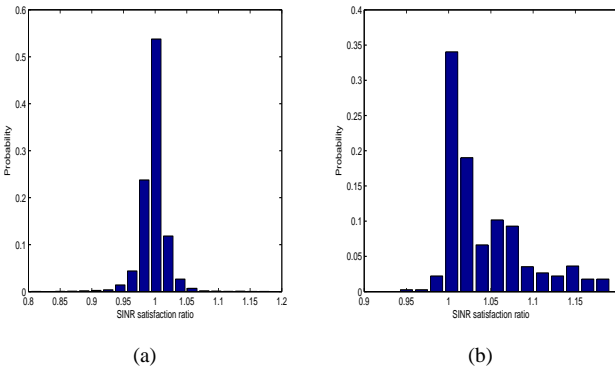


Fig. 2: Histograms of SINR satisfaction ratio for $\gamma = 10$ dB: a) non-robust scheme in [4], b) proposed robust strategy.

schemes, e.g., [7] and [3], become infeasible for high SINR requirements since they take no consideration of individual BS transmit power constraints in their problem formulation. Therefore, the proposed design is of practical significance, especially for dense users distribution since it optimally scales the SINR targets based on per BS power budgets and always provides a feasible solution at the scaled target SINR.

Let SINR satisfaction ratio be defined as the achieved SINR over the scaled target SINR of UT_{ik} , i.e., $\eta_{ik} = \frac{SINR_{ik}}{c_i \gamma_{ik}}$, where $\eta_{ik} \geq 1$ represents that the SINR requirement of UT_{ik} is satisfied. Fig. 2 compares the SINR satisfaction ratio at $\gamma = 10$ dB target SINR of the proposed decentralized robust scheme and its non-robust counterpart in [4]. One can observe that for the proposed distributed robust design that provides protection against channel uncertainties, almost all of the SINR requirements are satisfied. However, the actual SINR fails to satisfy the SINR requirements for approximately 50 percent of the cases for the non-robust design. Thus, the beamforming designs based on perfect CSI may be sensitive to the channel uncertainties in a practical scenario. In comparison with Fig. 1, the performance gap between robust and non-robust schemes

can be interpreted as the cost for guaranteeing the worst-case QoS at UTs, i.e., robustness to the imperfect CSI.

V. CONCLUSION

This paper studies a distributed robust approach for maximizing the weighted SINR targets at each UT in multicell interference networks. The problem is constrained to strict transmit power constraints at individual BSs in the presence of imperfect CSI. This problem is firstly mapped to an equivalent centralized sum-power minimization problem at individual BSs. Then the global-wise problem is decomposed into parallel subproblems via projected subgradient iterations to coordinate the ICI across the BSs. Finally, a distributed UCB algorithm is proposed based on the concept of multi-arm bandit to find a global optimal trade-off between the weighted SINR targets and the per-BS transmit power constraints. Our simulation results confirm the advantages of the proposed transmission strategy in providing larger SINR operation range and robustness to the channel uncertainties in a multicell scenario with realistic parameter setup.

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